Oxford Cambridge and RSA

Monday 27 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

1 **(a)** **(i)** Given that $f(x) = \arctan x$, write down an expression for $f'(x)$. Assuming that x is small, use a binomial expansion to express $f'(x)$ in ascending powers of x as far as the term in x^4. **[3]**

(ii) Hence express $\arctan x$ in ascending powers of x as far as the term in x^5. **[3]**

(b) Find, in exact form, the value of the following integral.

$$\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3-4x^2}}\,dx$$

[5]

(c) A curve has polar equation $r = \dfrac{a}{\sqrt{\theta}}$ where $a > 0$.

(i) Sketch the curve for $\dfrac{\pi}{4} \leqslant \theta \leqslant 2\pi$. **[2]**

(ii) State what happens to r as θ tends to zero. **[1]**

(iii) Find the area of the region enclosed by the part of the curve sketched in part (i) and the lines $\theta = \dfrac{\pi}{4}$ and $\theta = 2\pi$. Give your answer in an exact simplified form. **[4]**

2 **(a)** **(i)** Express $2\sin\frac{1}{2}\theta\left(\sin\frac{1}{2}\theta - j\cos\frac{1}{2}\theta\right)$ in terms of z where $z = \cos\theta + j\sin\theta$. **[3]**

(ii) The series C and S are defined as follows.

$$C = 1 - \binom{n}{1}\cos\theta + \binom{n}{2}\cos 2\theta - \ldots + (-1)^n\binom{n}{n}\cos n\theta$$

$$S = -\binom{n}{1}\sin\theta + \binom{n}{2}\sin 2\theta - \ldots + (-1)^n\binom{n}{n}\sin n\theta$$

Show that

$$C + jS = \left\{-2j\sin\tfrac{1}{2}\theta\left(\cos\tfrac{1}{2}\theta + j\sin\tfrac{1}{2}\theta\right)\right\}^n.$$

Hence show that, for even values of n,

$$\frac{C}{S} = \cot\left(\tfrac{1}{2}n\theta\right).$$

[8]

(b) Write the complex number $z = \sqrt{6} + j\sqrt{2}$ in the form $re^{j\theta}$, expressing r and θ as simply as possible.

Hence find the cube roots of z in the form $re^{j\theta}$.

Show the points representing z and its cube roots on an Argand diagram. **[7]**

3 **(i)** Find the eigenvalues and eigenvectors of the matrix **M**, where

$$\mathbf{M} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}.$$

Hence express **M** in the form $\mathbf{PDP}^{-1}$ where **D** is a diagonal matrix. **[8]**

(ii) Write down an equation for $\mathbf{M}^n$ in terms of the matrices **P** and **D**.

Hence obtain expressions for the elements of $\mathbf{M}^n$.

Show that $\mathbf{M}^n$ tends to a limit as n tends to infinity. Find that limit. **[6]**

(iii) Express $\mathbf{M}^{-1}$ in terms of the matrices **P** and **D**. Hence determine whether or not $(\mathbf{M}^{-1})^n$ tends to a limit as n tends to infinity. **[4]**

Section B (18 marks)

4 **(i)** Given that $y = \cosh x$, use the definition of $\cosh x$ in terms of exponential functions to prove that

$$x = \pm \ln\left(y + \sqrt{y^2 - 1}\right).$$

[5]

(ii) Solve the equation

$$\cosh x + \cosh 2x = 5,$$

giving the roots in an exact logarithmic form. **[5]**

(iii) Sketch the curve with equation $y = \cosh x + \cosh 2x$. Show on your sketch the line $y = 5$.

Find the area of the finite region bounded by the curve and the line $y = 5$. Give your answer in an exact form that does not involve hyperbolic functions. **[8]**

END OF QUESTION PAPER